Jefferson Township Public Schools

Summer Assignment for students entering AP Calculus BC

This assignment will count as a formative assessment and is due on the second class meeting of the school year.
In order to complete the curriculum before the AP Exam in May, it is necessary to do some preparatory work this summer. The summer assignment, or review packet, helps you to focus on the mathematical skills and content you will need to use in solving Calculus problems. These problems deal with skills and content that you studied in Pre-Calculus.

* If a calculator has been used, then you must set up what you entered into the calculator and what the calculator produced for you on your paper.* Try to only use a calculator when stated in the instructions. About 60% of the AP Exam does not allow use of a calculator. If decimal answers are given, round to the nearest thousandth.

At this level, doing homework is more than just getting the problems done. The problems should be a learning experience. Take your time and make sure you understand the concepts behind each problem. Seek out help to deal with problems and concepts you find challenging. I recommend that you try to meet with other AP Calculus BC students in small groups this summer to help each other. We are all in this together!

If you don’t know or remember how to do a problem, there are plenty of resources online, such as Khan Academy, Youtube and Purplemath. It is your responsibility to be prepared for the first day of class.

The AP Calculus BC curriculum is extensive considering it tests AB and BC material. Therefore, there are some pre-requisite topics you need to learn on your own over the summer. This helps us to focus on the calculus portion of these topics next year.
1. Find \( f(x + \Delta x) \) for \( f(x) = x^2 - 2x - 3 \).

2. Find \( \frac{f(x + \Delta x) - f(x)}{\Delta x} \) if \( f(x) = 8x^2 + 1 \).

3. Find \( \frac{f(x+h) - f(x)}{h} \) if \( f(x) = \frac{1}{x} \).

4. Given \( f(x) = x^2 - 3x + 4 \) find \( f(x + 2) - f(2) \).

5. Simplify each expression completely.
   
   (a) \( \frac{\sqrt{x}}{x} \)
   
   (b) \( e^{\ln 3} \)
   
   (c) \( \ln 1 \)

   (d) \( \ln e^7 \)
   
   (e) \( \log_{1/2} 8 \)

   (f) \( e^{3\ln x} \)

   (g) \( e^{\ln 4 + \ln 7} \)
   
   (h) \( \frac{x/2}{x/4} \)

   (i) \( \frac{3x(x+1) - 2(2x+1)}{(x-1)^2} \)

   (j) \( \frac{\sqrt{x^2} - 1}{\sqrt{x+2}} \)
   
   (k) \( \frac{\sqrt{x^2} + 5}{x-2} \)

   (l) \( \frac{2x}{x-1} + \frac{4}{x^2 - 4x + 3} \)
6. Condense $\frac{1}{2}\ln(x - 3) + \ln(x - 6) - 6 \ln x$

7. Evaluate $\log_2 5$ to the nearest thousandth.

8. Solve for $y'$: $xy' + y = 1 + y'$

9. Let $f(x) = \begin{cases} 3x + 2, & x < 2 \\ -x^2, & x \geq 2 \end{cases}$
   (a) find $f(1)$
   (b) find $f(5)$

10. Determine all points of intersection. Round to the nearest thousandth if necessary.
   (a) $y = x^2 + 3x - 4$
       $y = 5x + 11$
   (b) $y = x - 1$
       $y^2 = 2x + 6$
   (c) $y = x^3 - 2x^2 + x - 1$
       $y = -x^2 + 3x - 2$

11. For the functions below, give the zeros (if non exit, write none), domain, range, VA's, HA's, and/or points of discontinuity (holes-as ordered pairs) if any exist. Also sketch the functions graph.
   (a) $f(x) = \frac{x+3}{2x^2+5x-3}$
   (b) $f(x) = \frac{(x^2-3)(x+2)^2}{(x-3)(x+1)(4x^2+3)}$

**Trigonometry**

12. List the three Pythagorean identities

13. List the double angle formulas.
   (a) $\sin 2x =$
   (b) $\cos 2x =$
14. List the sum and difference formulas
(a) \( \cos(\alpha \pm \beta) = \)  
(b) \( \sin(\alpha \pm \beta) = \)

15. List the power reducing formulas
(a) \( \sin^2 x = \)  
(b) \( \cos^2 x = \)

Find the simplest exact value of each of the following.

16. \( \sin \frac{7\pi}{6} \)
17. \( \cos \left( -\frac{\pi}{3} \right) \)
18. \( \tan \frac{4\pi}{3} \)

19. \( \csc \frac{-5\pi}{4} \)
20. \( \sec \frac{5\pi}{6} \)
21. \( \cot \frac{2\pi}{3} \)

22. \( \sin^{-1} 0.5 \)
23. \( \sec^{-1} 2 \)
24. \( \cos^{-1} \frac{-\sqrt{3}}{2} \)

25. Simplify
(a) \( \sin^2 x + \cot^2 x \sin^2 x \)  
(b) \( \csc x - \cos^2 x \csc x \)  
(c) \( \frac{\sin^2 x + \sin x - 6}{\sin x + 3} \)

26. Find the solution of the equations for \( 0 \leq x \leq 2\pi \)
(a) \( 2 \sin^2 x = 1 - \sin \theta \)  
(b) \( 2 \tan \theta - \sec^2 \theta = 0 \)
(c) \( \sin 2\theta + \sin \theta = 0 \)

**Limits**

27. Find the limits, if they exist.

(a) \( \lim_{x \to 4} \frac{2x^3 - 7x^2 - 4x}{x - 4} \)

(b) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{9 - x} \)

(c) \( \lim_{x \to 1} \frac{x^2 - 2x - 5}{x + 1} \)

(d) \( \lim_{x \to 2} \frac{x^3 + 8}{x + 2} \)

(e) \( \lim_{x \to -2} \frac{x - 4}{x^3 - 2x - 8} \)

(f) \( \lim_{x \to 5} \frac{x - 5}{|x - 5|} \)

(g) \( \lim_{x \to 1} \frac{x - 1}{x^2 - 1} \)

(h) \( \lim_{x \to 5} 2x^2 - 3x + 4 \)

(i) \( \lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} \)
(j) \[ \lim_{{x \to -3}} \frac{x^2 + x - 6}{x^2 - 9} \]

(k) \[ \lim_{{x \to 25}} \frac{\sqrt{x - 5}}{x^2 - 25} \]

(l) \[ \lim_{{t \to \infty}} \frac{6t^2 + 5t}{(1-t)(2t-3)} \]

(m) \[ \lim_{{x \to \infty}} \cos x \]

(n) \[ \lim_{{x \to 0}} \frac{\sqrt{x+4} - 2}{x} \]

(o) \[ \lim_{{x \to \infty}} \frac{2x^2}{5x^2 - 9x - 2} \]

(p) \[ \lim_{{x \to \infty}} \frac{x^2 + x}{3 - x} \]

(q) \[ \lim_{{x \to -2}} \frac{x^3 + 8}{x + 2} \]

28. Explain why each function is discontinuous and determine if the discontinuity is removable or non-removable.

(a) \( g(x) = \begin{cases} 2x - 3, & x < 3 \\ -x + 5, & x \geq 3 \end{cases} \)

(b) \( b(x) = \frac{x(3x+1)}{3x^2 - 5x - 2} \)

(c) \( h(x) = \frac{\sqrt{x^2 - 10x + 25}}{x - 5} \)
29. Determine if the following limits exist, based on the graph below of \( p(x) \). If the limits exist, state their value. Note that \( x = -3 \) and \( x = 1 \) are vertical asymptotes.

(a) \( \lim_{x \to 1^-} p(x) \)  
(b) \( \lim_{x \to -3^-} p(x) \)

(c) \( \lim_{x \to 2} p(x) \)  
(d) \( \lim_{x \to 3^-} p(x) \)

(e) \( \lim_{x \to 3^+} p(x) \)  
(f) \( \lim_{x \to -1} p(x) \)

For #30-32 each of the following determine: a) \( \lim_{x \to 1^-} f(x) \)  
b) \( \lim_{x \to 1^+} f(x) \)  
c) \( \lim_{x \to 1} f(x) \)

30. \( f(x) = \begin{cases} x^2 - 1, & x < 1 \\ 4 - x, & x \geq 1 \end{cases} \)  
31. \( f(x) = \begin{cases} 3x - 1, & x \leq 1 \\ 3 - x, & x > 1 \end{cases} \)  
32. \( f(x) = \begin{cases} -x^2, & x < 1 \\ 2, & x = 1 \\ x - 2, & x > 1 \end{cases} \)

33. Use the graph of \( f(x) \), shown below, to answer the following questions.

(a) For what value of \( a \) is \( \lim_{x \to a} f(x) \) nonexistent?

(b) \( \lim_{x \to \infty} f(x) \)

(c) \( \lim_{x \to -\infty} f(x) \)
34. Use the graph below for the following questions.

(a) \( \lim_{x \to 3^-} f(x) \)  
(b) \( \lim_{x \to 4^+} f(x) \)  
(c) \( \lim_{x \to 3^+} f(x) \)  
(d) \( \lim_{x \to 4} f(x) \)  
(e) \( \lim_{x \to -3} f(x) \)  
(f) \( \lim_{x \to 6^-} f(x) \)  
(g) \( \lim_{x \to -6^+} f(x) \)  
(h) \( \lim_{x \to 2^-} f(x) \)  
(i) \( \lim_{x \to 6} f(x) \)  
(j) \( \lim_{x \to 2^+} f(x) \)  
(k) \( \lim_{x \to 2} f(x) \)  
(l) \( \lim_{x \to \infty} f(x) \)  
(m) \( \lim_{x \to 4^-} f(x) \)  
(n) \( \lim_{x \to \infty} f(x) \)  
(o) \( f(2) \)  
(p) \( f(3) \)  

35. Consider the function \( f(x) = \begin{cases} 
  x^2 + kx, & x \leq 5 \\
  5 \sin \left( \frac{\pi}{2} x \right), & x > 5
\end{cases} \). In order for the function to be continuous at \( x = 5 \), find the value of \( k \).

36. Consider the function \( f(x) = \begin{cases} 
  \frac{\sin x}{x}, & x \neq 0 \\
  k, & x = 0
\end{cases} \). In order for the function to be continuous at \( x = 0 \), find the value of \( k \).
Non-Traditional Composite/Operation Limits

The commonly memorized limit of composition rule generally is presented as something like this:

\[ \lim_{x \to a} f(g(x)) = f \left( \lim_{x \to a} g(x) \right) \]

provided that \( \lim_{x \to a} g(x) = L \) and \( f(x) \) is continuous at \( x = L \).

This is useful, but like many if \(-\) then statements it gets misread as an if and only if statement. If the \( f(x) \) is not continuous at \( x = L \) or if the limit of \( g(x) \) does not exist at \( x = a \), then we cannot apply the property. It does not tell us anything conclusive about the existence of \( \lim_{x \to a} f(g(x)) \).

Similar discussions follow from all the 'standard' limit properties. E.g., If \( \lim_{x \to a} f(x) = L_1 \) and \( \lim_{x \to a} g(x) = L_2 \), then \( \lim_{x \to a} (f(x) + g(x)) = L_1 + L_2 \). However, if \( \lim_{x \to a} f(x) \) and/or \( \lim_{x \to a} g(x) \) don't exist, we cannot draw any conclusion about \( \lim_{x \to a} (f(x) + g(x)) = L_1 + L_2 \).

What can we do when the traditional limit properties don't apply? Well they also can be applied to one-sided limits. The most direct and effective route to exploring the limit is to remember another property of limits that IS an if and only if statement:

\[ \text{Iff } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L \text{, then } \lim_{x \to a} f(x) = L. \]

By exploring the right-hand and left-hand compositions or operations, we can often find if the limit being explored exists or not.

Consider:

<table>
<thead>
<tr>
<th>the graph of ( f(x) )</th>
<th>the graph of ( g(x) )</th>
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Find \( \lim_{x \to 1} g(x) \) \( \lim_{x \to 2} [f(x) \cdot g(x)] \) \( \lim_{x \to 0} f(x) \)

(a) For this composition, traditional rules apply.

(b) While the limit of \( f(x) \) doesn't exist at \( x = 2 \), explore the left-hand and right-hand limits of the products.

(c) It is important to consider from what direction the inner limit value is approaching 2. [Hint: Sometimes a tabular approach helps see what's happening, even if you don't know the actual function.]

Find \( \lim_{x \to -1} [f(x) + g(x)] \).

Find \( \lim_{x \to 1} [f(x) + g(x + 1)] \).
1. Find \( f(g(4)) \).

2. Find \( \lim_{x \to 4} f(g(x)) \).

3. Find \( f(g(0)) \).

4. Find \( \lim_{x \to 0} f(g(x)) \).

5. Find \( \lim_{x \to 6^-} g(1 - f(x)) \).

6. Find \( \lim_{x \to 3^+} (15 - x^2) \).

7. Find \( \lim_{x \to 2^-} (2 - x^2) \).

8. Find \( \lim_{x \to -2^-} f(f(x)) \).
I. Sketch the graph of each function & state the domain and range of each:

A) $y = x$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

B) $y = x^2$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

C) $y = x^3$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

D) $y = \frac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
E) \( y = \sqrt{x} \)

\[ \text{Domain: } \quad \text{Range: } \]

F) \( y = e^x \)

\[ \text{Domain: } \quad \text{Range: } \]

G) \( y = \ln x \)

\[ \text{Domain: } \quad \text{Range: } \]

H) \( y = \sin x \)

\[ \text{Domain: } \quad \text{Range: } \]

I) \( y = \tan x \)

\[ \text{Domain: } \quad \text{Range: } \]

J) \( y = \tan^{-1} x \)

\[ \text{Domain: } \quad \text{Range: } \]
K) $y = |x|$

L) $y = \sqrt[3]{x}$

Domain: Domain:
Range: Range:
Transcendental Functions: Trig, Exponential, and Logarithmic Functions

Trig Facts: You will need to know these as you know your multiplication tables. The values for sin, cos, and tan in Quadrant I should be *memorized cold*. The other Quadrants’ values should not take you more than a few seconds to say when quizzed. Every trig function takes an angle as input and returns a ratio as output. So, every inverse trig function takes a ratio as input and returns ________ as output.

I. Fill out the following 16 point unit circle by finding the following:

1) The measures for each angle in radian and in degree.
2) The coordinate pair for each angle.

II. Find all values of $x$ (to the nearest thousandth) which make the statement true (Use a calculator to solve).

A) $-\cos 2x = e^{-x^2}, -\pi \leq x \leq \pi$
III. Simplify each expression into a real number without a calculator.

\[ A) \ln \left( \sqrt[3]{e^2} \right) = \]
\[ C) e^{2 \ln 4} = \]
\[ E) \ln \left[ \sin^2 \left( \frac{13\pi}{7} \right) + \cos^2 \left( \frac{13\pi}{7} \right) \right] = \]
\[ G) \sec^2(2.1) - \tan^2(2.1) = \]

\[ B) \tan^{-1} \left( \frac{-\sqrt{3}}{3} \right) = \]
\[ D) e^{2 + \ln 4} = \]
\[ F) \ln \left| -1 \right| = \]

IV. Re-write each as a single logarithm or trig function.

\[ A) \ln(x + 2) + \ln(x - 4) - \ln(x - 3) = \]
\[ B) \cos^2(x + 5) - \sin^2(x + 5) = \]
\[ C) \frac{\ln(x)}{\ln(3)} = \]

V. Simplify without a calculator

\[ A) \frac{100!}{97!} \quad B) \frac{(n + 1)!}{(n - 1)!} \]